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Optimal reactive power control of DGs for voltage regulation of MV distribution systems using sensitivity analysis method and PSO algorithm

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ABSTRACT

This paper addresses the problem of reactive power control of distributed generation (DG) units in the medium voltage (MV) distribution systems to maintain the system voltages within the predefined limits. An efficient approach for the load flow calculation is used here which is based on the topological structure of the network. It has been formulated for the radial distribution systems. A direct voltage sensitivity analysis method is developed in this paper which is also based on the topological structure of the network and independent of the network operating points. Thus, the sensitivity matrix is calculated once with the load flow program and it is used in all the system working conditions. The problem of DGs reactive power control is formulated as an optimization problem which uses the sensitivity analysis for linearizing the system around its operating points. The objective of the optimization problem is to return the system voltages inside the permitted range by using the reactive power of DGs in an optimal way. The optimal solutions are obtained by implementing particle swarm optimization (PSO) algorithm. Then, the results are verified by running a load flow considering new values of DGs reactive power. The procedure is repeated as long as a voltage violation is observed. Simulation results reveal that the proposed algorithm is capable of keeping the system voltages within the permitted limits.

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Introduction

During the recent years, the conventional structure of electrical power systems has been changed by the presence of distributed generation units (DGs). Previously, the electric power was generated in large generating stations at a small number of locations (called central generation). In these stations, voltage was stepped up to high voltage (HV) to be transmitted through HV transmission networks. The voltage was then stepped down to medium voltage (MV) and low voltage and distributed through radial distribution networks to the end users. In recent years, there has been a considerable growth in the amount of decentralized generation connected to the distribution systems. As a result, currently, power systems are in a state of transition from the conventional systems with unidirectional power flows to the active networks with bidirectional power flows. Therefore, new technical challenges have emerged for distribution system operators (DSOs) [1]. Presence of distributed generation units alters the conventional voltage regulation schemes of distribution systems. In the conventional distribution feeder (without DG units), voltage decreases toward end of the feeder, as the impedance of lines causes a voltage drop. Thus, the biggest voltage drop happens at the end of the feeder based on the amount of loads demand. With the presence of DG, if its power exceeds the local demand of loads, the power flow direction will be inversed and a voltage rise will appear at the DG-connected bus.

Traditionally, DSOs have managed their system at the planning stage based on the fit and forget policy using deterministic load flow studies (considering the critical cases) in order to meet the loads demand and to verify lines capacity and voltage regulation issues. With DG units, as their output power varies during the day, the uncertainty in distribution system management is increased and the safe operation of the system becomes more complicated. In this situation, implementing an on-line voltage control system based on the active network management policy becomes more crucial.

Theoretically, different methods can be applied for voltage regulation of distribution systems but the most applicable methods







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are based on using on-load tap changer (OLTC) mechanism of the transformer, reactive power compensation and curtailment of DG active power. In [1], coordination of the OLTC action and the reactive power changes provided by distribution STATCOM has been studied. The problem of reactive power control of DGs for voltage regulation of the radial distribution systems by using a new voltage sensitivity analysis method has been addressed in [2]. The proposed algorithm acts on a single generator that has the biggest effect on the voltage of the violated bus. A method for voltage regulation of MV distribution systems based on the generation curtailment of DGs active power has been presented in [3]. The generation curtailment is done using the voltage sensitivity factors extracted from Jacobian matrix. Also, an algorithm for short term scheduling of distribution systems including day-ahead scheduler and intra-day control system has been developed in [4]. It is formulated as a non-linear optimization problem that is linearized by the use of the sensitivity coefficients obtained from the load flow calculations. A centralized voltage control scheme based on the model predictive control (MPC) and using the sensitivity indexes has been investigated in [5]. A coordinated scheme to minimize the cost of the system operation in terms of cost of the energy losses, cost of the curtailed energy and cost of the reactive power support while maintaining the system limits has been formulated as an optimal power flow (OPF) algorithm in [6]. In [7], a hierarchical three-layer control scheme has been developed for the control of active and reactive powers of DGs using the MPC and OPF algorithms. Authors in [8] have presented a new method called the experimental design method for the optimal positioning, sizing and eventually for the real time control of the system voltages in the MV grids.

In the literature, in [3–6], the sensitivity matrix extracted from Jacobian matrix is used which is not constant and it changes with respect to the network operating point. Also, the sensitivity method based on the Jacobian matrix suffers from inaccuracies because it should incorporate the variation of load powers with voltage, which is not well known in practice [5]. In this paper, a direct approach is presented to obtain the voltage sensitivity coefficients based on the topological structure of the network. The proposed sensitivity method is independent of the network operating points and has a simple theory and structure. Finally, in this work, a centralized voltage control method is developed to optimally manage the reactive power of DG units using PSO algorithm and the proposed sensitivity approach. The main objective of this work is to return the system voltages inside the permitted limits while minimizing the reactive power changes of DG units.

Voltage control problem in the new distribution systems

The electric power networks have traditionally been operated in a passive mode where the power generated by large power plants was delivered to the customers through distribution networks. Thus, the flow of power was from the higher toward the lower voltage levels. Recently, in the emerging electric networks, distributed generation units are expected to play an increasing role. With the introduction of the DGs, the power flows may be reversed. The distribution network is no longer a passive circuit supplying loads but an active system with the power flow and voltage determined by the DGs as well as the loads. When a DG injects active power at a certain point of the system, the voltage of that node can be raised. This fact is explained as follows. Consider the radial system shown in Fig. 1.

In case of no DG, the power flow between nodes 1 and 2 (P_{12} and Q_{12}) is equal to the load demand at bus 2 ($P_L + jQ_L$). The voltage drop in per unit at bus 2 can be given approximately by [9,10]

$$V_1 - V_2 = P_{12}r_{12} + Q_{12}x_{12} \tag{1}$$



Fig. 1. Part of a distribution system.

where r_{12} and x_{12} are the resistance and reactance of the line between nodes 1 and 2. Now, if the DG injects power at node 2, the power flow between nodes 1 and 2 is changed. In this case, (1) must be modified as

$$V_1 - V_2 = (P_L - P_{\rm DG})r_{12} + (Q_L \pm Q_{\rm DG})x_{12}$$
⁽²⁾

where P_{DG} and Q_{DG} are active and reactive powers of the DG unit. Based on (2), when active power of the DG increases, the term P_L – P_{DG} can become sufficiently negative so that the right side of (2) becomes negative that means V_2 is greater than V_1 . So the injection of DG power can cause a voltage rise problem, especially, when the x/r ratio is low that is the case in the distribution systems. As it can be observed, the amount of voltage variations depends on the amount of DGs active and reactive powers, demand of loads and impedance of the system lines. Due to the fact that demand of loads and DGs active power are changing during the day, both voltage rise and voltage drop problems are possible to occur. The voltage control problem is known as one of the biggest obstacles for increasing the integration of DG units in distribution grids. If this problem can be solved efficiently, then higher DG levels could be allowed to be installed on the feeders.

Reactive power compensation

Reactive power compensation is a useful method for voltage regulation of distribution systems. Traditionally in distribution systems, capacitor banks have been used to keep the power factor close to 1 and to compensate voltage drop in the heavy load situations. In the DG-connected distribution systems, as we must deal with both voltage drop and voltage rise problems, we need a source of reactive power with the ability to work in both inductive and capacitive modes (see (2)).

The needed reactive power of the system can be provided by synchronous machine-based DG units that are able to adjust their output reactive power in order to affect the system voltages. Conventional control systems for reactive power control of synchronous machines are automatic power factor control (APFC) system and automatic voltage regulation (AVR) system [11]. In the automatic power factor control mode, the reactive power of DG follows any variation of the active power of DG. Therefore, the P_{DG}/Q_{DG} ratio is maintained constant in order to keep the system voltage within the limits. This method is not applicable in voltage regulation of distribution systems with a low ratio of x/r. Also, it is not an effective approach as the load power variations of the system are not taken into consideration. In the automatic voltage control mode, the difference between the actual bus voltage and a set reference voltage defines the needed reactive power of the system. This action can be explained by a droop characteristic that shows the relationship of the needed reactive power of DG in accordance with the voltage of the system. In [12], a new voltage control method has been proposed which combines the advantages of AVR and APFC control systems. The above-mentioned reactive power control methods are based on the local control of DGs unit.

In case of doubly-fed induction generators (DFIGs), reactive power compensation is possible through control of rotor current but the physical, thermal and converter power limitations must be considered. DFIG-based wind turbines are able to control active and reactive powers independently. In this paper a centralized approach is proposed to remotely control reactive power of DGs.

Direct approach for load flow study in distributions systems

The idea of direct load flow approach in the distribution systems has been presented in [13]. This approach is based on developing two matrices named bus injection to branch current (BIBC) and branch current to bus voltage (BCBV). These matrices present the topological structure of the network. The BIBC matrix is responsible for the relations between the bus current injections and branches current. Thus, variations of the branches current, which are generated by the variations of the current injection at the system buses, can be found directly from the BIBC matrix. The BCBV matrix presents the relations between the branches current and system voltages. The variations of the system voltages caused by the variations of the branches current can be found by using the BCBV matrix. A simple 5-bus distribution system shown in Fig. 2 is used to explain the theory of the proposed load flow study.

The branches current B_1 , B_2 , B_3 and B_4 are obtained by applying the Kirchhoff's current law to the system in Fig. 2:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix}$$
(3)

The BIBC matrix that gives the relationship between branches current and bus current injection at the system buses is obtained from (3) as

$$[B] = [BIBC][I] \tag{4}$$

The relationships between branches current and system voltages can be presented by the following equation.

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 \\ Z_{12} & Z_{23} & 0 & Z_{35} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$$
(5)

The BCBV matrix can be obtained from the over-mentioned equation.

$$\Delta V] = [BCBV][B] \tag{6}$$



Fig. 2. Simple 5-bus radial distribution system.

Using (4) and (6), a direct relation between ΔV and *I* is obtained through the so-called DLF matrix that is composed by multiplying BCBV and BIBC matrices.

$$[\Delta V] = [BCBV][BIBC][I] = [DLF][I]$$
(7)

The DLF matrix is built once in load flow study. It remains constant as it contains the topological structure of the network. In order to solve the load flow, (7) is used in an iterative-based procedure. The vectors of ΔV and I are updated by using the following equations.

$$I_i^k = \left(\frac{P_i + jQ_i}{V_i^k}\right)^* \tag{8}$$

$$[\Delta V^{k+1}] = [\text{DLF}][I^k] \tag{9}$$

$$[V^{k+1}] = [V^0] - [\Delta V^{k+1}] \tag{10}$$

where V^k is the vector of the system voltages at the iteration number k. Vector V^0 assumes all the initial voltage values equal to 1 per unit. The solution of load flow is obtained when the error between the new calculated voltage values (V^{k+1}) and V^k is less than the chosen error (for all the system buses).

In this load flow method, time-consuming LU decomposition and forward/backward substitution of the Jacobian matrix or admittance matrix required in the traditional load flow methods are no longer necessary [13]. Only the DLF matrix is used to solve the load flow problem. Therefore, in this method, a considerable computation time can be saved that makes the proposed load flow method suitable for online operation and management of the system.

The proposed voltage sensitivity analysis method

The selection of the optimum control strategy is made by using off-line data based on the sensitivity analysis. Obviously, changing all system parameters has an effect on the system performance. However, some may have significant impacts whereas others have less important impacts. The voltage sensitivity matrix provides us some information about the influence of changing generation and load parameters (*P* and *Q*) on the system voltages. Voltage sensitivity of each node to the injection of reactive or active power of the system nodes is usually obtained from the inverse of Jacobian matrix in the load flow study as shown in (11).

$$\begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$
(11)

where $\Delta \theta$ is vector of voltage angle variations, ΔV is vector of nodal voltage variations and J^{-1} is the inverse of the Jacobian matrix. The drawback of using this method is that the sensitivity coefficients extracted from Jacobian matrix is not constant and it changes with respect to the network operating point [3]. Recalculating and updating these data are time-consuming and increase calculation burden of the proposed algorithm. Moreover, as stated before, in MV distribution systems when DGs inject power to the grid, the system voltages tend to increase. In HV transmission grids, as the resistance of lines is negligible this phenomenon happens mainly when reactive power is injected to the system and injection of active power mostly affects the voltage angles of the systems. In other words, that so called *Pθ*–*QV* decoupling of HV grids is no longer valid in MV grids. The sensitivity method based on the Jacobian matrix is generally valid in MV distribution grids but in our application, we do not mind about the voltage angle values. The objective of voltage control system is to maintain the magnitude of the system voltages within the limits. Thus, this method is a complex and time-consuming approach to be used in MV systems. In order to tackle the shortcomings of the classical sensitivity analysis method, a direct approach for calculation of the sensitivity matrix independent of the network operating points and well-suited for MV distribution systems is presented.

From (2) presenting the voltage variations at bus 2 with respect to the voltage at bus 1 (slack bus), it can be observed that the active power that flows in the line is coupled with the resistance of line and the reactive power that flows in the line is coupled with the reactance of line. The voltage variations at bus 2 are function of the reactive and active powers of the line, but the influence degree of P_{12} and Q_{12} on voltage at bus 2 depends on the r_{12} and x_{12} , respectively. In a general form, (2) can be written as

$$V_i - V_j = r_{ij}P_{ij} + x_{ij}Q_{ij} \tag{12}$$

where P_{ij} and Q_{ij} are the active and reactive power flows between nodes *i* and *j* ($i \neq j$). As the voltage at bus 1 is always constant, the sensitivity of system voltages to the active and reactive powers in each branch can be obtained from the resistance and reactance of the relevant lines, respectively. In order to make the proposed sensitivity analysis method clearer, let us refer again to the simple 5bus radial distribution system shown in Fig. 2. Eq. (12) for that test case can be rewritten as:

$$\begin{bmatrix} \Delta V_{12} \\ \Delta V_{13} \\ \Delta V_{14} \\ \Delta V_{15} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} r_{12} & 0 & 0 & 0 \\ r_{12} & r_{23} & 0 & 0 \\ r_{12} & r_{23} & 0 & r_{35} \end{bmatrix} \begin{bmatrix} P_{12} \\ P_{23} \\ P_{34} \\ P_{35} \end{bmatrix} + \begin{bmatrix} x_{12} & 0 & 0 & 0 \\ x_{12} & x_{23} & 0 & 0 \\ x_{12} & x_{23} & 0 & 0 \\ x_{12} & x_{23} & 0 & x_{35} \end{bmatrix} \begin{bmatrix} Q_{12} \\ Q_{23} \\ Q_{34} \\ Q_{35} \end{bmatrix}$$
(13)

Due to the fact that we aim to control the reactive power of the system at some nodes, the sensitivity of system voltages with respect to the power injection at the system nodes is needed. The relations between power flow in branches with the power injection at the system buses are obtained as below.

$$\begin{bmatrix} P_{12} \\ P_{23} \\ P_{34} \\ P_{35} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$
(14)

The same manner is used to build the matrix giving the relations between reactive power flow in branches with the reactive power injection at the system buses. Using (13), (14) and its reactive counterpart, we obtain the sensitivity of system voltages with respect to the active and reactive power injections at the system buses.

$$\begin{bmatrix} V_{1} \\ V_{1} \\ V_{1} \\ V_{1} \\ V_{1} \end{bmatrix} - \begin{bmatrix} V_{2} \\ V_{3} \\ V_{4} \\ V_{5} \end{bmatrix} = \begin{bmatrix} r_{12} & r_{12} & r_{12} & r_{12} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} + r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} + r_{12} + r_{23} + r_{12} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{13} & r_{12} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{13} & r_{12} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{12} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{13} & r_{12} & r_{12} + r_{23} \\ r_{13} & r_{12} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{13} & r_{12} & r_{12} + r_{23} & r_{12} + r_{23} \\ r_{13} & r_{12} & r_{12} + r_{12} \\ r_{14} & r_{14} & r_{14} & r_{14} \\ r_{15} & r_{15} \\ r_{15} &$$

In a general form, (15) can be written as

$$[\Delta V] = [R][P] + [X][Q]$$
(16)

where [P] and [Q] are vectors of the active and reactive power injections at the system nodes, respectively. Also, the matrices [R] and

[X] give the sensitivity of the system voltages with respect to the active and reactive powers of the system nodes. The element (m,n) of [R] is the sum of the resistance of the branches in which both P_{m+1} and P_{n+1} flow. For instance, in order to obtain the element (3,2) of [R], the branch between nodes 1 and 2 as well as the branch between nodes 2 and 3 in which both P_4 and P_3 flow are considered [2]. Since bus 1 is the slack bus, its voltage is always constant $\left(\frac{\partial V_1}{\partial P_1} = \frac{\partial V_1}{\partial Q_1} = 0\right)$. Therefore, the element (3,2) of [R] $(R_{3,2})$ that gives the sensitivity of voltage at bus 4 with respect to the active power at bus 3 is given by:

$$\frac{\partial(V_1 - V_4)}{\partial P_3} = \frac{-\partial V_4}{\partial P_3} = R_{3,2} = r_{12} + r_{23}$$
(17)

All the sensitivity indexes are obtained by the following rules.

$$\frac{\partial V_m}{\partial P_n} = -R_{m-1,n-1} \tag{18}$$

$$\frac{\partial V_m}{\partial Q_n} = -X_{m-1,n-1} \tag{19}$$

The proposed sensitivity analysis is developed based on the topological structure of the network. The sensitivity coefficients are built by considering the directions in which *P* and *Q* flow. The matrix *R* (or *X*) presents the relationship between the active (or reactive) power injection of the nodes and voltage of the nodes. The same approach has been used to develop DLF matrix in the load flow study. The DLF matrix is built in the same way of matrices *R* and *X*. It presents the relationships between [ΔV] and [*I*]. DLF contains the same data as matrices *R* and *X*. The only difference is that DLF matrix contains resistance and reactance with together in each array in the complex form:

$$[\mathsf{DLF}] = [R] + j[X] \tag{20}$$

Therefore, in this work, the voltage sensitivity coefficients are obtained from DLF matrix in the load flow study. The real part of DLF matrix in each array composes the matrix *R* and its imaginary part gives the matrix *X*.

Problem formulation

In this work, active power control of DGs is not considered, thus, from (16), the matrix *X* is used only. Also, reactive power control at all system buses is not possible. Therefore, in (15), the elements that correspond to the DG-connected buses are only employed.

The proposed load flow algorithm is used here to calculate the current state of the system and to check if there is voltage violation in the system. The sensitivity matrix ([X]) is used to calculate the proper change of DGs reactive power in order to manage the voltage constraints. As matrix X is constant and independent of network operating point, the expected amount of voltage change can be determined by calculating the proper amounts of reactive power changes. Using the sensitivity matrix, the problem is linearized around its operating points, so the current amount of DGs reactive power is not considered in this part of the problem. The main aim is to calculate the proper amount of reactive power changes.

The flowchart of the developed algorithm for optimal control of DGs reactive power is presented in Fig. 3. The main objective of the problem is to return the system voltages inside the permitted range when the voltage limits are violated by using reactive power control of DGs. Also, the amount of reactive power changes must be selected optimally. Therefore, the problem is formulated as an optimization problem with the relevant constraints. The objective



Fig. 3. Flowchart of the proposed algorithm.

function is chosen in a way that aims to minimize the total amount of reactive power changes.

Minimize objective function :
$$\sum_{x=1}^{N} |\Delta Q_{DGx}|$$
(21)

where *N* is the number of the DG units that contributes in reactive power control. After running the initial load flow program, if there is a voltage violation in the system, the bus with the biggest voltage violation is selected. Then, the amount of the biggest voltage violation from either upper or lower limit is calculated (ΔV_{worst}). It gives us the proper value of voltage change in order to return the voltage of that bus inside the permitted range (ΔV_{proper}). Form (15), the row corresponding to the bus with the worst voltage violation is used to calculate the proper amount of voltage change in order to solve the voltage problem of that bus as follows

$$\Delta V_{\text{proper}} = -\Delta V_{\text{worst}} = \sum_{x=1}^{N} \frac{\partial V_{\text{worst}}}{\partial Q_{\text{DGx}}} \Delta Q_{\text{DGx}}$$
(22)

where $\frac{\partial V_{\text{worst}}}{\partial Q_{\text{DCx}}}$ is the sensitivity of the voltage of the worst bus to the reactive power change of the DG number *x*. In (22), the sensitivity coefficients are constant variables which are obtained from matrix *X*. The proper amount of voltage change (ΔV_{proper}) for solving the voltage problem of the worst bus is a defined parameter but ΔQ_{DCx} (*x* = 1, 2, ..., *N*) are unknown variables that must be optimally selected. In order to consider the proper amount of voltage change

for solving the voltage problem of the worst bus, (22) is considered as an equality constraint of the problem.

In practice, the available capacity of DFIGs and synchronous machines for reactive power compensation is limited by several factors as presented in [14,15]. The DGs reactive power limitations are applied to the problem as an inequality constraint.

$$Q_{DGx,min} \leqslant Q_{DGx} \leqslant Q_{DGx,max}$$
(23)

By using (23), the power capability curve of the DG units is considered in the problem. Based on the performed problem formulation, the proposed algorithm aims to optimally change DGs reactive power to return the system voltages inside the permitted limits considering their available capacities. As it can be seen in Fig. 3, the iterative procedure of the algorithm stops when voltage of all the system buses is returned inside the permitted range.

Moreover, it must be noted that inside the main loop of the proposed algorithm shown in Fig. 3, load flow and particle swarm optimization algorithms are also iterative-based methods.

Particle swarm optimization (PSO) algorithm

PSO algorithm, introduced by Kennedy and Eberhart [16], is one of the modern heuristic algorithms that has the robustness to solve the optimization problems with continues and discrete variables. The PSO was developed through behavior of the natural society such as fishes and birds. It can generate a high quality solution within a shorter calculation time and more stable convergence characteristic than other methods [17]. In this paper, PSO algorithm is used to solve the optimization problem that was presented in the previous section as it has found to be a fast and robust optimization tool. The superiority of the PSO method to other evolutionary and search-based methods has been investigated in many papers in the recent years [18–21].

In this optimization algorithm, a group of particles is considered. Each particle has a position in the *N*-dimension space where *N* is the number of variables that must be optimized according to (21). Firstly, the algorithm starts with some random positions. For each particle, the proposed objective function is calculated by considering its position. Then, vector of Pbest is composed. It presents the best position of all particles. Also, the best particle of Pbest gives the gbest (the best agent of the group). At the next iteration, position of each particle is moved in order to find the better positions. The current position of each particle p at iteration *K* (v_p^k) combines with the distances to Pbest_p and gbest to find the new velocity of each particle. Mathematical formulation of this concept is shown as below.

$$\nu_p^{K+1} = w \times \nu_p^K + c_1 \times \text{rand}_1 \times \left(\text{Pbest}_p - S_p^K\right) + c_2 \times \text{rand}_2$$
$$\times \left(\text{gbest} - S_p^K\right)$$
(24)

where

 v_p^{K+1} is modified velocity vector of agent p; Pbest_p: best position of the particle p; S_p^K : position of agent p at iteration K; rand₁, rand₂: random numbers between 0 and 1; w: weighting factor; c_1, c_2 : learning factors = 2.

As can be seen in (24), direction and length of the new velocity vector for each particle is determined by combining its previous velocity with Pbest_p and gbest. Using (24), a new velocity vector is calculated and then the current position of each particle is modified by

$$S_{p}^{K+1} = S_{p}^{K} + v_{p}^{K+1} \tag{25}$$

At each iteration, if the new position of the particle is better than the previous one (based on the value of its objective function), Pbest_p is updated. Also, if the current gbest is better than that of the previous iteration, it should be updated as well. In this way, the particles are moving toward the better positions. PSO algorithm stops when it reaches the maximum iteration number (Iter_{max} = 200). If the optimization algorithm works correctly, at the end, all particles are stopped at the same position where is the best position. There are 10 particles in the group.

Investigated system

In order to test the effectiveness of the proposed algorithm, a 33-bus, 12.6 kV radial distribution system shown in Fig. 4 is considered. Parameters of the investigated system are given in [22]. Total active and reactive powers of the system loads are 3.72 MW and 2.3 Mvar, respectively. The system under study also consists of 4 DG units which are located at the buses 6, 12, 18 and 33. The DG units are DFIG-based type and identical with a maximum rated power of 1 MW.

The capability power curve of the DG units is obtained from [14]. In this paper, the capability curve is linearized by the points given in Table 1. Also, it is approximated as a symmetrical curve. As it can be understood, the maximum reactive power is function



Fig. 4. 33-bus radial distribution system.

 Table 1

 Power capability curve of DGs (% in DG rated power).

Point	Р	Q _{max} /Q _{min} (%)
1	0	±95
2	25%	±95
3	50%	±90
4	100%	+60

of the current active power value of the machine. It is supposed that all the points inside the capability curve presented in Table 1 can be provided by the machine.

In the load flow study, bus number 1 is considered as slack bus and all other buses are P-Q buses. DG units are modeled as the negative loads. Positive values of reactive power correspond to the inductive mode and the negative values show the capacitive range. The upper and lower permitted limits of voltages for all system buses are 1.03 and 0.97 pu, respectively [23].

Simulation results

The proposed algorithm shown in Fig. 3 is coded in MATLAB environment. It is applied to the radial distribution system shown in Fig. 4 in order to test the effectiveness of the results. In this paper the worst case scenarios in voltage regulation of the studied test case are simulated and the results are presented. Obviously, when the proposed algorithm succeeds to manage the voltage problem of the worst cases, it can manage all other system working conditions.

Case 1: full load – minimum generation

In this case, all the system loads are at their maximum power values and DG active power generations are 15% of the rated power. It can be estimated that at the end of the lines, voltage drop can occur. Fig. 5 presents the results of the load flow study for this test case when DG units only generate active power (without reactive power compensation). Using the proposed algorithm for reactive power control of DGs, the system voltages return to the permitted range as can be seen in Fig. 5.

Based on the proposed algorithm, after running the initial load flow (at I = 0), if a voltage violation is found in the system, the main loop of the proposed algorithm starts (I = 1). At iteration number one (I = 1), the voltage of the bus with the biggest voltage violation is defined. Then, the row that corresponds to that bus is taken from (15) and is used as the equality constraint of the optimization problem. The proper amount of reactive power changes in order to solve the voltage violation of that bus is obtained by PSO algorithm. Finally, at the end of the iteration number one, a new load flow calculation considering the new values of DGs reactive power is run to verify the obtained results and to check whether all the system voltages are within the limits or not. If a new voltage violated bus is found, iteration number two starts. The bus with the biggest voltage violation is defined again and the row that corresponds to that bus is selected from (15). A new optimization problem must be solved for voltage problem at that bus which is



Fig. 5. System voltages along the feeder in case 1.

Table 2Reactive power changes of DGs in case 1.

	<i>I</i> = 1	<i>I</i> = 2	<i>I</i> = 3
ΔV_{worst} At bus no	-0.0414 pu 17 0	–0.033 pu 32 –490 83 kvar	–0.0027 pu 30 –103 57 kvar
ΔQ_{DG2} ΔQ_{DG3} ΔQ_{DG4}	0 -693.7 kvar 0	0 0 -950 kvar	-205.62 kvar 0 0
42.04			

subjected to the new equality constraint and the remained capacity of DGs reactive power. At the end of the iteration number two, load flow calculation again determines the iterative procedure must go to the next iteration or it can stop. The proposed algorithm stops when all the system buses are returned inside the permitted voltage range. Table 2 gives the values of DGs participation in voltage control of the system in the case 1.

The negative value of $(\Delta V_{\text{worst}})$ shows the amount of the biggest voltage violation from the lower voltage limit (0.97 pu). The negative values of ΔQ mean changing the reactive power toward the capacitive mode of the capability curve.

Let us now describe the results in Table 2 in more details. By running the initial load flow calculation (at I = 0), it is understood that there is voltage violation at the system buses. Thus, the main loop of the proposed algorithm starts with the iteration number one (I = 1). The biggest voltage violation is found at bus 17. At I = 1, the optimization algorithm employs only DG₃ (at bus 18) to regulate the voltage at bus 17. Because the sensitivity of voltage at bus 17 with respect to the reactive power change at bus 18 is the highest (among other DGs). Also, as the available capacity of DG₃ is enough to solve the voltage problem at bus 17, it is used alone. If it was not enough, the next DG candidate was DG₂. Finally, the last options were DG₁ and DG₄. It can be justified by the fact that

$$\left|\frac{\partial V_{17}}{\partial Q_{18}}\right| > \left|\frac{\partial V_{17}}{\partial Q_{12}}\right| > \left(\left|\frac{\partial V_{17}}{\partial Q_6}\right| or \left|\frac{\partial V_{17}}{\partial Q_{33}}\right|\right)$$

At the second iteration (I = 2), the proposed algorithm must solve the voltage drop at bus 32. As DG₄ has the biggest effect on voltage at bus 32 (among DG₁, DG₂ and DG₃), it is used by optimization algorithm with its maximum available reactive power capacity. Also, the rest of needed reactive power is provided by DG₁. Finally, at I = 3, the optimization problem must solve the voltage drop that happens at bus 30. Since DG₄ cannot be participated anymore, DG₁ and DG₂ are used. DG₁ and DG₂ have equal sensitivity to voltage at bus 30. The illustrated results in Fig. 5 with the red¹ dotted line show the system voltages at the end of the iteration number 3 when the proposed algorithm stops as all system voltages are returned to the predefined limits.

Case 2: maximum generation - minimum load

In the second case, the loads and DGs are considered to be 25% and 90% of their respective rated values. Fig. 6 shows the system voltages after and before the reactive power compensation by DGs.

As it can be seen, firstly, the biggest voltage violation is found at bus 18. Based on the proposed algorithm when the voltage problem at bus 18 is solved, then the new violated bus (bus 33) forms a new optimization problem with the new constraints. Table 3 presents the reactive power changes of DG units in order to regulate the system voltages. Considering the results of Table 3, the same analysis as the previous test case can be done here. At the first iteration of the main algorithm (I = 1), DG₂ and DG₃ are contributing at their maximum values. Since their contribution is not enough to return the voltage at bus 18 inside the permitted limits, DG₁ is also participated. Then, next iteration of the algorithm is required since there are still voltage violations in the system. The biggest voltage violation happens at bus 33. At the second iteration (I = 2), DG₂ and DG₃ cannot provide reactive power anymore (as they are already participated at their maximum values). So the only options are DG₁ and DG₄. Due to the fact that the sensitivity of voltage at bus 33 with respect to the reactive power change at bus 33 is higher than that of the bus 6 (for DG₁), DG₄ is used only to regulate the voltage at bus 33.

Discussion on the results

As noted before, inside the main loop of the algorithm shown in Fig. 3, load flow and particle swarm optimization algorithms are also iterative-based methods. The convergence of load flow program depends on the system parameters. If the input data of the load flow program are acceptable, its convergence is guaranteed. Since in this paper, the reactive power limitation of DG units is taken into account, the input data of load flow algorithm are adopted with the real constraints of the system. So the optimization algorithm will not demand some big values of reactive power changes that can cause load flow program to be diverged. PSO algorithm stops when it reaches the maximum iteration of the algorithm. Generally, the main convergence criterion for the whole algorithm is to have enough reactive power capacity in order to be participated in the voltage regulation problem.

In the proposed voltage sensitivity method, an approximation is considered in formulation of (1). As in the distribution systems voltage angles (Θ_i) are very small, the imaginary part of the voltage variation vector that corresponds to $\sin \theta_i$ is negligible. Therefore, (1) is obtained from the real part of the voltage variation vector. Based on the simulation results, it can be concluded that this is a very good and accurate approximation. Because the predicted values using the sensitivity method (in Tables 2 and 3) always match the calculated values obtained from the load flow program (in Figs. 5 and 6). For instance, it is seen in Fig. 5 that by using the proposed algorithm, voltage at bus 30 reaches 0.97 pu at the end of iteration 3. Also, in case 2, in Fig. 6, the system voltage at bus 33 reaches 1.03 pu. It means that the values obtained by the sensitivity method match the calculated values by the load flow program.

The execution time of the whole algorithm (shown in Fig. 3) in case 1 is 0.223 s and for the case 2 is 0.135 s by using a normal desktop computer (processor: core i5, 3.1 GHz, RAM: 4 GB). The notable speed of the algorithm comes from the fact that the proposed load flow method is developed for the distribution systems and it does not need the time-consuming procedure of the classical load flow methods. Also, separating load flow and optimization algorithm in the main loop of the proposed algorithm helps to not use all system data in the optimization problem. In fact, by the use of the sensitivity indexes, only the effects of the controllable variables on system voltages are employed in the optimization part of the proposed algorithm.

In the proposed algorithm, the parameters of loads and DGs (*P* and *Q*) needed for load flow study are not available in the practical cases. In order to adapt the proposed algorithm with the realistic cases, we can replace the load flow program with some limited voltage measurements on the critical buses (like buses 18 and 33 in this work). Because the objective of the load flow program is to find the voltage violations in the system and to validate the results after the corrective action of DGs. Thus, the limited voltage measurements can provide us these data instead of using the load

 $^{^{1}\,}$ For interpretation of color in Fig. 5, the reader is referred to the web version of this article.



Fig. 6. System voltages along the feeder in case 2.

Table 3Reactive power changes of DGs in case 2.

	<i>I</i> = 1	<i>I</i> = 2
$\Delta V_{ m worst}$	+0.066 pu	+0.0023 pu
At bus no	18	33
$\Delta Q_{\rm DG1}$	+480 kvar	0
$\Delta Q_{\rm DG2}$	+660 kvar	0
ΔQ_{DG3}	+660 kvar	0
$\Delta Q_{ m DG4}$	0	+68.4 kvar

flow program. The amount of active and reactive powers of each DG unit must be provided by the measurement as well. Using these data, the proposed algorithm can have the voltage of the critical buses as well as the current power and available reactive power capacity of DGs. Therefore, we can establish a closed-loop voltage control system by using the data from the measurement, sensitivity analysis and the optimization algorithm. In this way, the proposed algorithm is adapted for the realistic cases.

Conclusion

In this work an algorithm based on the sensitivity analysis was proposed for voltage regulation of MV distribution systems. The proposed algorithm is designed to find the optimal contribution of DGs reactive power in order to return the system voltages inside the permitted limits. An efficient load flow method based on the topological structure of the grid and well-suited for distribution systems is used to validate the results. Also, a direct sensitivity analysis method based on the topological structure of the network is presented. Thanks to the similarities between the load flow and sensitivity analysis methods, the sensitivity indexes are built at the same time with the load flow program.

The problem was formulated as an optimization problem which aims to minimize the reactive power changes of DG units while returning the voltage of the violated buses inside the permitted limits with taking into account the available reactive power capacity of DGs. The developed algorithm always gives the priority of the voltage regulation to the bus with the biggest voltage violation. If there is more than one voltage violated node in the studied system, the procedure can be repeated to regulate the voltage of other buses as well. Simulation results reveal that the proposed algorithm is capable of returning system voltages inside the permitted range while using the reactive power of DGs in an optimal way. Thanks to the fast convergence speed of the proposed algorithm, it can be used in on-line voltage management of the MV distribution systems including DG units. In the future research, it is planned to add the active power curtailment of DG units to the objective function of the optimization problem. By using active power control of DGs, we can have a more complete and flexible voltage control system. Also, we can give the priority to first use the reactive power or active power in order to regulate voltage.

References

- Bakhshideh Zad B, Lobry J, Vallée F. Coordinated control of on-load tap changer and D-STATCOM for voltage regulation of radial distribution systems with DG units. In: 3rd International conference on electric power and energy conversion systems, Istanbul, Turkey, October 2013.
- [2] Brenna M, De Beradinis E, Foiadelli F, Sapienza G, Zaninelli D. Voltage control in smart grid: an approach based on sensitivity analysis. J Electromagn Appl 2010:467–74.
- [3] Zhou Q, Bialek JW. Generation curtailment to manage voltage constraints in distribution networks. IET Gener Transm Distrib 2007;1(3).
- [4] Borghetti A, Bosetti M, Grillo S, Paolone M, Silvestro F. Short-term scheduling of active distribution systems. In: 2009 IEEE power tech conference, Bucharest, Romania; 2009.
- [5] Valverde G, Van Cutsem T. Model predictive control of voltages in active distribution networks. IEEE Trans Smart Grid, Special Issue on Optimization Methods and Algorithms Applied to Smart Grid 2013.
- [6] Pilo F, Pisano G, Soma GG. Optimal coordination of energy resources with a two-stage online active management. IEEE Trans Industr Electron 2011;58(10):4526–37.
- [7] Farina M, Guagliardi A, Mariani F, Sandroni C, Scattolini R. Model predictive control of voltage profiles in MV networks with distributed generation. Control Eng Pract 2015;34:18–29.
- [8] Toubeau JF, Vallée F, De Grève Z, Lobry J. A new approach based on the experimental design method for the improvement of the operational efficiency in medium voltage distribution networks. Electr Power Energy Syst 2015;66:116–24.
- [9] Scott NC, Atkinson DJ, Morrell JE. Use of load control to regulate voltage on distribution networks with embedded generation. IEEE Trans Power Syst 2002;17(2).
- [10] Elkhatib ME, El-Shatshat R, Salama MMA. Novel coordinated voltage control for smart distribution networks with DG. IEEE Trans Smart Grids 2011;2(4):598–605.
- [11] Vovos PN, Kiprakis AE, Wallace AR, Harrison GP. Centralized and distributed voltage control: impact on distributed generation penetration. IEEE Trans Power Syst 2007;22(1):476–83.
- [12] Kiprakis AE, Wallace AR. Hybrid control of distributed generators connected to weak rural networks to mitigate voltage variations. CIRED 2003.
- [13] Teng Jen-Hao. A direct approach for distribution system load flow solutions. IEEE Trans Power Delivery 2003;18(3):882–7.
- [14] Engelhardt S, Erlich I, Feltes C, Kretschmann J, Shewarega F. Reactive power capability of wind turbines based on doubly fed induction generators. IEEE Trans Energy Convers 2011;26(1).
- [15] Adibi M, Milanicz D. Reactive capability limitation of synchronous machines. IEEE Trans Power Syst 1994;9(1):29–40.
- [16] Kennedy J, Eberhart R. Particle swarm optimization. In: IEEE international conference on neural networks; 1995. p. 1942–8.
- [17] Hasanvand H, Bakhshideh Zad B, Mozafari B, Feizifar B. Damping of lowfrequency oscillations using an SVC-based supplementary controller. IEEJ Trans Electr Electron Eng 2013;8(6):550–6.

- [18] Devia S, Geethanjalib M. Optimal location and sizing of distribution static synchronous series compensator using particle swarm optimization. Electr Power Energy Syst 2014;62:646–53.
- [19] Bakhshideh Zad B, Hasanvand H, Lobry J, Maskani H, Mozafari B. Improvement of dynamic response of AGC system using thyristor controlled phase shifter and optimization of system parameters. Int Rev Electr Eng 2012;7(6):6182–9.
- [20] Kansal S, Kumar V, Tyagi B. Optimal placement of different type of DG sources in distribution networks. Electr Power Energy Syst 2013;53:752–60.
- [21] Yoshida H, Kawata K, Fukuyama Y, Takayama S, Nakanishi Y. A particle swarm optimization for reactive power and voltage control considering voltage security assessment. IEEE Trans Power Syst 2000;15(4):1232–9.
- [22] Baran ME, Wu Felix F. Network reconfiguration in distribution systems for loss reduction and load balancing. IEEE Trans Power Delivery 1989;4(2).
- [23] Masters CL. Voltage rise the big issue when connecting embedded generation to long 11 kV overhead lines. Power Eng J 2002;16(1):5–12.